Lecture 3

- Review (Friedmann equations)
- Redshift
- Luminosity distance
- Parallax

$\left.\begin{array}{l}\text{Redshift}\\ \text{Luminosity distance}\\ \text{Parallax}\end{array}\right\}$ measure of times and distances in the U.

Friedmann equations

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \Lambda = -8\pi G p$$

$$\frac{\partial}{\partial t}\left(\rho a^3\right) + p\,\frac{\partial a^3}{\partial t} = 0 \implies$$

$$\implies \rho_i = a^{-3(1+\omega_i)} \cdot C \quad , \quad p_i = \omega_i \rho$$

Some new notation: $\frac{\dot a}{a} \equiv H$ (Hubble parameter)

$$H^2 = \frac{8\pi G}{3}\left(\sum \rho_i\right)$$

$i = \Lambda, m, \gamma, k \ldots$

$$\rho_c \equiv \frac{3H_0^2}{8\pi G}$$

$$\Omega_i \equiv \frac{\rho_{i,0}}{\rho_c} \quad , \quad \sum \Omega_i \equiv 1$$

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_\gamma\left(\frac{a_0}{a}\right)^4\right]$$

If we multiply by a we get

$$\dot{a}^2 = U(a) \sim \text{potential motion}$$

- We discussed solutions where one of the components dominates as well as global dS ($\Lambda + k$)

FRW metric:

$$ds^2 = -dt^2 + a^2(t)\frac{d\sigma^2}{1-kr^2} + a^2(t)\, d\Omega_2^2$$

We would like to understand how to measure distances in cosmology and get some more intuition about FRW spacetimes.

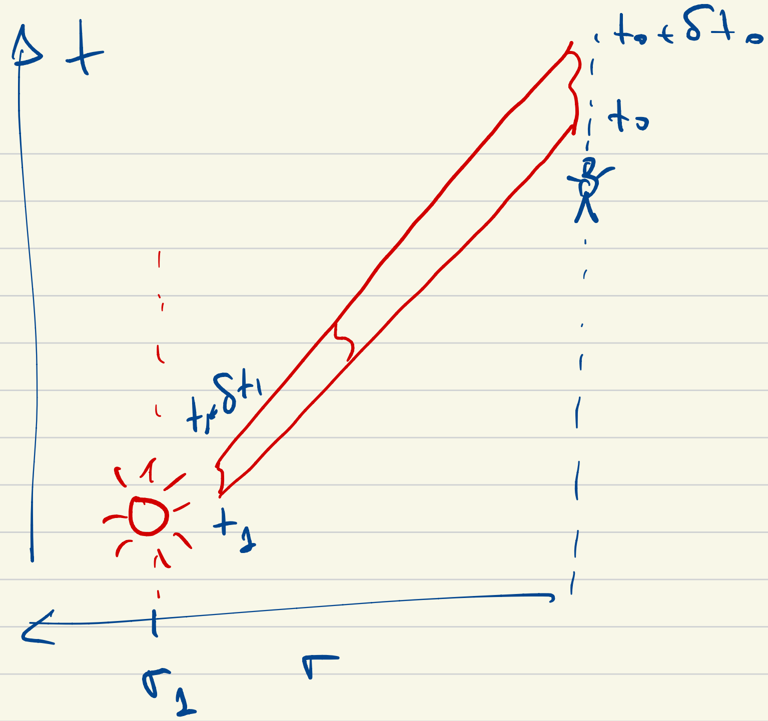
Redshift:

Light travels along the null geodesics. Let us suppose Ω_2 angles are constant. and light is emitted at $r = r_1$ at $t = t_1$ and absorbed by us at $r = 0$ at $t = t_0$

$$dt = -a(t) \frac{dr}{\sqrt{1-kr^2}}$$

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \arcsin r_1 & k>0 \\ r_1 & k=0 \\ \operatorname{arcsinh} r_1 & k<0 \end{cases}$$

$$\overset{\text{def}}{=} f(r)$$



second pulse

at $t = t_1 + \delta t_1$.

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)} \Rightarrow$$

$$\Rightarrow \frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)}$$

$$\delta t \sim \frac{1}{\omega} \Rightarrow \omega_0 = \omega_1 \frac{a(t_1)}{a(t_0)}$$

if the universe is expanding

$$\omega_0 < \omega_1 \quad (\lambda_0 > \lambda_1)$$

the light got redshifted!

redshift $z = \dfrac{\lambda_0 - \lambda_1}{\lambda_0} = \dfrac{a(t_0)}{a(t_1)} - 1$

Redshift is often used as measure of time / distance, Because we can measure it as we know λ's of many transitions.

- Physical distance vs coordinate distance

$$l_p = a(t)\, l_c$$

$$\frac{dl_p}{dt} = \dot{a}\, l_c = \frac{\dot{a}}{a}\, l_p = H l_p$$

[Hubble law]

Now $H = H_0 \approx \frac{70 \text{ km/s}}{\text{Mpc}}$ $\quad \text{Mpc} = 3 \cdot 10^{22}\,\text{m}$

$1\,\text{pc} = 3\,\text{l.y.} = 3 \cdot 10^{16}\,\text{m}$

We would like to measure distances to faraway objects. Two main methods are:

Luminocity distance and Paralax

Luminocity distance

- Standard candle ~ object of known luminocity. In the absence of expansion apparent luminosity is

$$P = L\frac{S}{4\pi d^2}$$

$S \to$ telescope area

$d \equiv$ luminocity distance, in general.

Let us see what d depends on in expanding universe:

$$P = L\underbrace{\left(\frac{S}{S_{tot}}\right)}_{\substack{\text{total}\\ \text{area of}\\ \text{the light front}}}\underbrace{\left(\frac{\hbar\omega_0}{\hbar\omega_1}\right)}_{\substack{\text{redshift}\\ E\sim\hbar\omega}}\underbrace{\left(\frac{\delta t_1}{\delta t_0}\right)}_{\substack{\text{time}\\ \text{interval}\\ \text{changed}}}$$

$$S_{tot} = 4\pi r_1^2 a^2(t_0)$$

$$P = L \frac{a^2(t_1)}{a^2(t_0)} \frac{S}{4\pi a^2(t_0) r^2} \Rightarrow$$

$$\Rightarrow d = \frac{a^2(t_0)}{a(t_1)} r(t_0, t_1) = r(t_0,t_1)\, a(t_0)(1+z)$$

Now we want to express r. h. s. just terms of z. This will determine evolution of $a(t)$, if we measure both z and d.

remember that

$$r(t_0,t_1) = \begin{cases} \sin \displaystyle\int_{t_1}^{t_0} \frac{dt}{a(t)} & k=1 \\ \displaystyle\int_{t_1}^{t_0} \frac{dt}{a(t)} & k=0 \\ \sinh \displaystyle\int_{t_1}^{t_0} \frac{dt}{a(t)} & k=-1 \end{cases}$$

Now we need to find $\displaystyle\int_{t_1}^{t_0} \frac{dt}{a(t)}$

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_\gamma\left(\frac{a_0}{a}\right)^4\right]$$

define $\quad x = \dfrac{a}{a_0}$, $\quad A^2(x) = \Omega_\Lambda + \Omega_k \dfrac{1}{x^2} + \Omega_m\dfrac{1}{x^3} + \Omega_\gamma\dfrac{1}{x^4}$

$$\frac{dx}{dt} = H_0 A(x)\cdot x \quad \Rightarrow$$

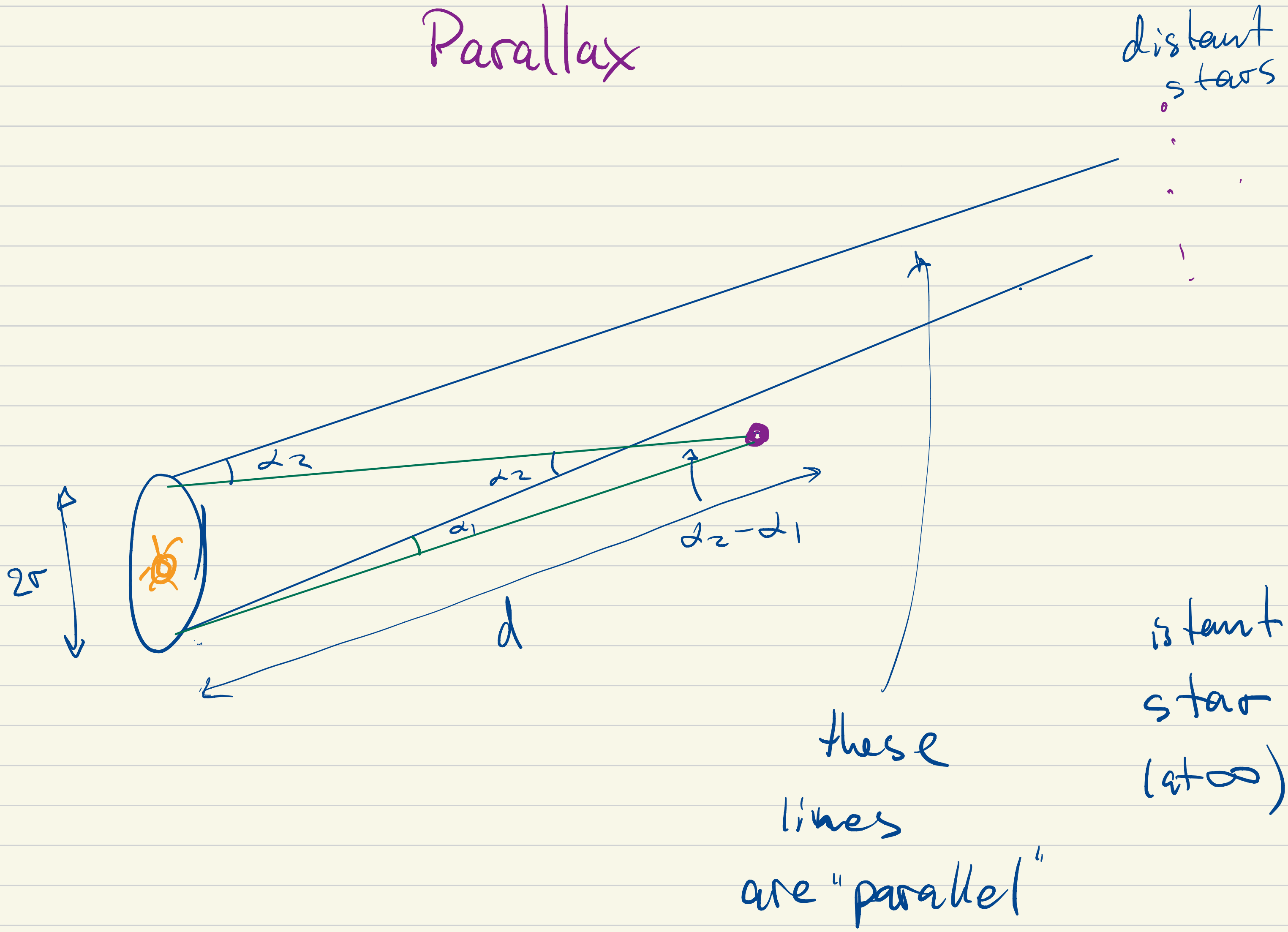
$$\Rightarrow \int_{t_1}^{t_0}\frac{dt}{a(t)} = \int_{t_1}^{t_0}\frac{dt}{dx}\frac{dx}{a_0 x} = \frac{1}{a_0 H_0}\int_{\frac{1}{1+z}}^{1}\frac{dx}{x^2 A(x)}$$

Finally:

$$d(z) = a_0(1+z)\frac{1}{\sqrt{-k}}\sinh\frac{\sqrt{-k}}{a_0 H_0}\int_{\frac{1}{1+z}}^{1}\frac{dx}{x^2 A(x)}$$

depends on matter content

Parallax



$$2r = d(\alpha_2-\alpha_1)$$

$$d = \frac{2r}{\alpha_2-\alpha_1}$$

$1''$ parallax $\Rightarrow$ $d =$ parsec

Gaia $\sim 10^{-5} \cdot 1''$

$\sim 10^{4\div5}$ l.y. dist.